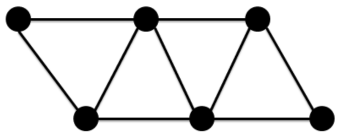
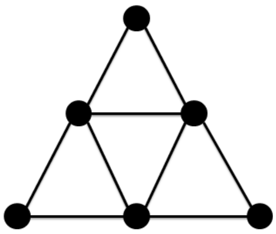
JMO Spatial Reasoning Questions

Level: Junior   Ref No: J04   Puzz Points: 14

[JMO 2002 B4] (i) Network A has nine edge edges with meet at six nodes. The numbers 1, 2, 3, 4, 5, 6 are placed at the nodes, with a different number at each node. Is it possible to do this so that the sum of the two numbers at the ends of an edge is different for each edge? Either show a way of doing this, or prove that it is impossible to do so.



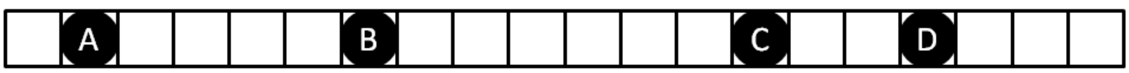
(ii) Repeat the same procedure for Network B, i.e. show that it is possible to place the six numbers so that the sum of the two numbers at the ends of an edge is different for each edge, or prove that it is impossible to do so.



Solution: (i) Possible (ii) Impossible

Level: Junior   Ref No: J06   Puzz Points: 15

[JMO 2002 B6] A game for two players uses four counters on a board which consists of a 20 x 1 rectangle. The two players take it in turns to move one counter. A turn consists of moving any one of the four counters any number of squares to the right, but the counter may not land on top of, or move past, any of the other counters. For instance, in the position shown, the next player could move D one, two or three squares to the right, or move C one or two squares to the right and so on.



The winner of the game is the player who makes the last legal move. (After this move the counters will occupy the four squares on the extreme right of the board and no further legal moves will be possible.)

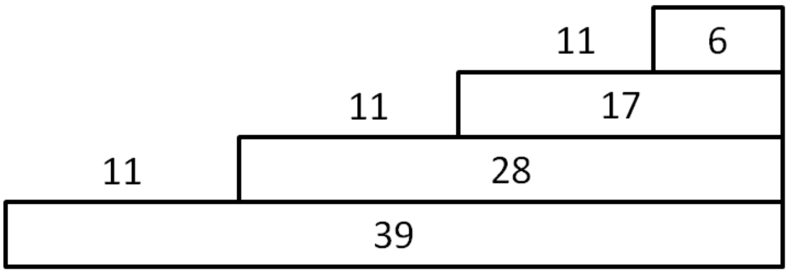
In the position shown above, it is your turn. Which move should you make and what should be your strategy in subsequent moves to ensure that you will win the game?

Solution: Strategy is to make gap between A and B the same number of squares (possibly 0) as the gap between C and D.

Level: Junior Ref No: J07 Puzz Points: 10

[JMO 2000 B1] Kate has 90 identical building blocks. She uses all of the blocks to build this four-step 'staircase' in which each step, apart from the top one, is the same length.

i) Show that there are exactly two different ways in which it is possible to use all 90 blocks to build a six-step 'staircase'.
ii) Explain fully why it is impossible to use all 90 blocks to build a seven-step 'staircase'.

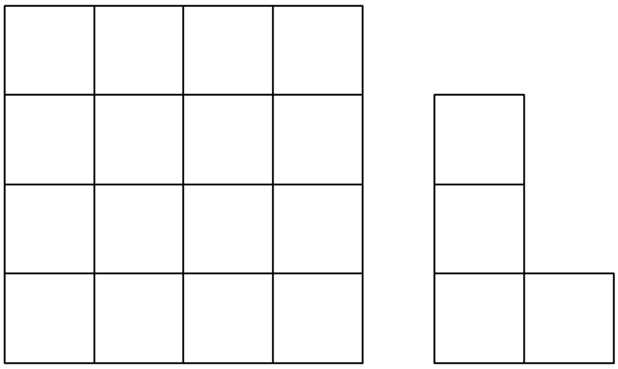


Solution: (i) 10, 12, 14, 16, 18, 20 and 5, 9, 13, 17, 21, 25 (ii) Possible to construct algebraic expression with a factor of 7, but 90 is not a multiple of 7.

Level: Junior Ref No: J18 Puzz Points: 15

[JMO 2007 B6] We want to colour red some of the cells in the 4 x 4 grid shown so that wherever the L-shaped piece is placed on the grid it covers at least one red cell. The L-shaped piece may only cover complete cells, may be rotated, but may not be turned over and may not extend beyond the grid.

(a) Show that it is possible to achieve this by colouring exactly four cells red.
(b) Show that it is impossible to achieve this by colouring fewer than four cells red.



Solution: (a) If numbering cells 1 to 16 across each row, shade 3, 7, 10, 14. (b) Suppose L shapes joined in pairs to form two 4 by 2 rectangles. If fewer than 4 cells coloured then at least one L will have no cells coloured.

Level: Junior Ref No: J23 Puzz Points: 14

[JMO 2006 B5] An intelligent bug starts at the point (4,0) and follows these instructions:

(i) First face "East" and walk one unit to the point (5,0);
(ii) From then one, whenever you arrive at a point (x,y) with x and y both integers,
either turn left through 90 if x – y is a multiple of 4 or is 1 more than a multiple of 4;
or turn right through 90 if x – y is 2 more than a multiple of 4 or is 3 more than a multiple of 4;
and then walk one unit to the next point whose coordinates are both integers.

After one move, the bug is at the point (5,0).

(a) Where will the bug be after 12 moves?
(b) Where will the bug be after 50 moves?

Solution: (a) (6, 2), and facing East (b) (13, 9)

Level: Junior Ref No: J30 Puzz Points: 15

[JMO 2001 B6] This question is about ways of placing square tiles on a square grid, all the squares being the same size. Each tile is divided by a diagonal into two regions, one black and one white. Such a tile can be placed on the grid in one of four different positions as shown below.

When two tiles meet along an edge (side by side or one below the other) the two regions which touch must be of different types (i.e. one black and one white).

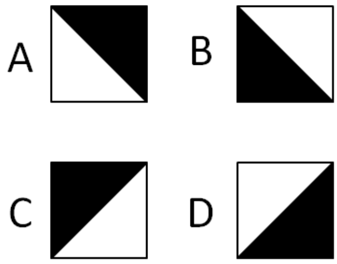
(i) A 2 x 2 grid of four squares is to be covered by four tiles.

(a) If the top-left square is covered by a tile in position A, find all the possible ways in which the other three squares may be covered.

(b)In how many different ways can a 2 x 2 grid be covered by four tiles?

(ii) In how many different ways can a 3 x 3 grid be covered by nine tiles?

(iii) Explaining your reasoning, find a formula for the number of different ways in which a square grid measuring n x n can be covered by n^2 tiles.

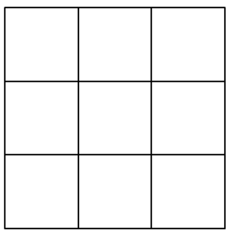


Solution: (i) (a) 4 (b) 1 (ii) 64 (iii) 4^n (or 2^{2n})

Level: Junior Ref No: J36 Puzz Points: 15

[JMO 2011 B6] Pat has a number of counters to place into the cells of a 3 x 3 grid like the one shown. She may place any number of counters in each cell or leave some of the cells empty. She then finds the number of counters in each row and each column. Pat is trying to place counters in such a way that these six totals are all different.

What is the smallest total number of counters that Pat can use?



Solution: 8 (you need to prove $n \geq 8$ to ensure there wasn't a solution using fewer counters, and you also need to show an arrangement using 8 counters)

Level: Junior Ref No: J37 Puzz Points: 10

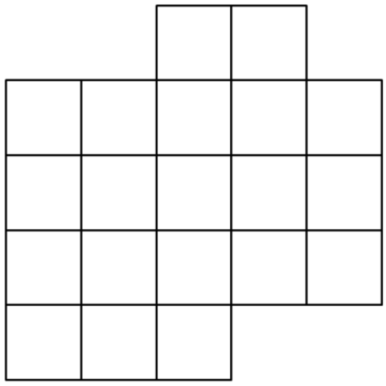
[JMO 2008 B1] Tamsin has a selection of cubical boxes whose internal dimensions are whole numbers of centimetres, that is, 1cm x 1cm x 1cm, 2cm x 2cm x 2cm, and so on.

What are the dimensions of the smallest of these boxes in which Tamsin could fit ten rectangular blocks each measuring 3cm x 2cm x 1cm without the blocks extending outside the box?

Solution: A 4cm cube

Level: Junior   Ref No: J59   Puzz Points: 14

[JMO 2009 B5] An ant wishes to make a circuit on the board shown, visiting each square exactly once and returning to the starting square. At each step the ant moves to an adjacent square across an edge. Two circuits are considered to be the same if the first follows the same path as the second but either starts at a different square or follows the same path in reverse. How many circuits are possible?



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